

PHIL 4310: Advanced Logic
Spring 2026
Homework 5

This homework is due Fri, April 10th.
Read Chapter 4.1-4.4 of *Logic for Philosophy*.

Homework questions:

Part I)

Do exercise 4.1 (page 143).

Do exercise 4.2 (page 153). So two parts. First, assume that $\Box\phi =_{\text{def}} \sim\Diamond\sim\phi$. Now show that the truth conditions for $\Box\phi$ are as given on page 150 by using the truth-conditions for \sim and diamond. And then analogously if diamond is defined as $\sim\Box\sim\phi$.

Do exercise 4.3 (page 153). Further hint: If there is no chain of the accessibility relation R from w_0 to w_n , then the truth values of any formulas at w_n (and indeed the very existence of w_n) cannot affect the truth value of any formulas at w_0 .

Do exercise 4.5 d, e, g, j, l, n (page 183)

Part II)

1) Give an argument that no formula of the form $\Box\Diamond\phi$ is valid in K.

2) Prove that for any formula ϕ , $\Box\phi$ is valid in modal logic D iff $\Diamond\phi$ is valid in D iff ϕ is valid in D. In other words, either all three of $\Box\phi$, $\Diamond\phi$, and ϕ are valid in D or none are.

NOTE: This is NOT the same thing as claiming that $\Box\phi$, $\Diamond\phi$, and ϕ are all equivalent in D. In fact, none of $\Box\phi \leftrightarrow \Diamond\phi$, $\Box\phi \leftrightarrow \phi$, $\Diamond\phi \leftrightarrow \phi$ are valid in D.